

# FPGA Based Gaussian Pulse Shaping Filter Using Distributed Arithmetic Algorithm

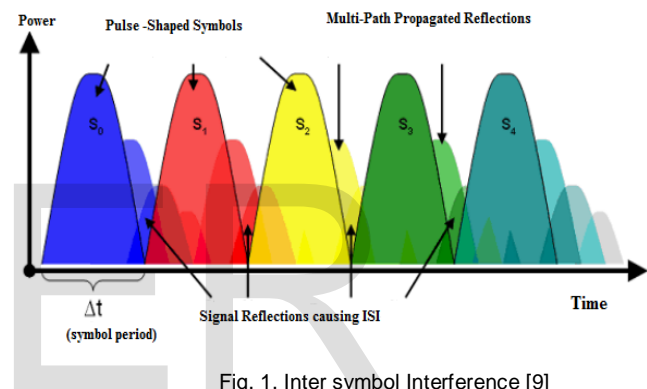
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**Abstract**— Pulse shaping is utilized to increase transmission data rate without increasing the bandwidth or bit error rate. In this paper an efficient approach is presented to design and implement a high speed Gaussian pulse shaping filter for ISI removal. The proposed filter has been designed and simulated using Matlab, synthesized with Xilinx Synthesis Tool (XST), and implemented on Spartan 3E based 3s500efg320-5 FPGA device using DALUT algorithm. The developed Gaussian filter structure can operate at an estimated frequency of 116.306 MHz by utilizing 4% of Slices, 1% of Slice Flip Flops and 4 % LUTs of target FPGA device. Hence, high-speed multiplier less design of pulse shaping FIR filter is achieved.

**Keywords**—DALUT, FPGA, GSM, GMSK, HDTV, MATLAB, XST

## 1 Introduction

In a digital communication system, digital information can be sent on a carrier through changes in its fundamental characteristics such as: phase, frequency, and amplitude. In a physical channel, these transitions can be smoothed, depending on the filters implemented in transmission. Filters are an important part of communications systems. They are used to eliminate spectral leakage, reduce channel width, and eliminate interference from adjacent symbols (Inter Symbol Interference). In today's environment, however, digital transmission has become a much more challenging proposition. The main reason is that the number of bits that must be sent in a given time interval i.e. data rate is continually increasing. Unfortunately, the data rate is constrained by the bandwidth available for a given application. Furthermore, the presence of noise in communications system also puts a constraint on the maximum error-free data rate. The wide diffusion of wireless terminals like cellular phones is opening new challenges in the field of mobile telecommunications. Besides, the possibility to transmit not only voice but even data between terminals and end users of many kinds has fostered the development of new technologies and new standards for cellular communications [1]. As digital technology ramps up for this century, an ever-increasing number of RF applications will involve the transmission of digital data from one point to another. The general scheme is to convert the data into a suitable baseband signal that is then modulated onto an RF carrier [2]. Transmitting signals at a high modulation rate through a band-limited channel causes Inter symbol Interference.



In electronics and telecommunications, **pulse shaping** is the process of changing the waveform of transmitted pulses.

The purpose of pulse shaping is to make the transmitted signal better suited to its purpose for the communication channel, typically by limiting the effective bandwidth of the transmission. It helps in minimizing sharp transitions at the input causing the frequency components of the output to remain within a specified frequency range. This enables the transmission of signals using lower transmission power. The shapes of the pulses should be chosen such that they do not interfere with each other at the optimal sampling point. To ensure this the pulse shape should decay rapidly outside the pulse interval and should exhibit zero crossing at all intervals except its own. Recently, there is increasingly strong interest on implementing multi-mode terminals, which are able to process different types of signals, e.g. WCDMA, GPRS, WLAN and Bluetooth. These versatile mobile terminals favor simple receiver architectures because otherwise they'd be too costly and bulky for practical applications [3].

In this work, a digital FIR filter design was chosen for implementing the pulse shaping filter. FIR filters were chosen over IIR digital filters as they are more stable and can be designed to

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have linear phase and require no feedback. Digital filters are typically used to modify attributes of signal in the time and frequency domain through the process called linear convolution. This process is formally described by the following formula.

$$y[n] = x[n] * f[n] = \sum_k x[k].f[n - k]$$

$$= \sum_k x[k].c[k] \quad (1)$$

where  $c[k]$  are filter's coefficients. When the filter coefficients do not change over time, then digital filters are generally classified as being FIR or IIR filters. The output of an FIR filter of order or length  $N$  to an input time sample  $x[n]$  is given by a finite version of convolution sum:

$$y[n] = \sum_{k=0}^N x[k].c[k] \quad (2)$$

Equation (2) implies that the FIR filter is realized by a large number of adders, multipliers and delay elements. Pulse shaping filters are normally implemented as oversampled finite impulse response (FIR) digital filters. Gaussian filters are significantly better than RRC filters [4]. Gaussian FIR filters give significantly better BER performances than the conventional RRC filter. The FIR structure with linear phase technique is efficient as it takes advantage of symmetrical coefficients and uses half the required multiplications and additions [5], [6].

## 2 Shaping Filters

In communications systems, two important requirements of a wireless communications channel demand the use of a pulse shaping filter. The first requirement is generating band limited channels, and the second requirement is reducing Inter Symbol Interference (ISI) arising from multi-path signal reflections.

Both requirements can be accomplished by a pulse shaping filter which is applied to each symbol. Pulse shaping filter are often used in communication transmitters for baseband processing in order to improve the transmission efficiency of a signal spectrum. The pulse shaping filters are widely used in Mobile Phones, HDTV, Space communication, Radar, Audio/data/CD/video system, Speech synthesis recognition, A/D and D/A conversion.

The pulses are sent by the transmitter and these are detected by the receiver in any data transmission system. At the receiver, the goal is to sample the received signal at an optimal point in the pulse interval to maximize the probability of an accurate binary decision. This implies that the fundamental shapes of the pulses be such that

they do not interfere with one another at the optimal sampling point.

Before digital filters were available, pulse shaping was accomplished with analog filters

There are two criteria that ensures non interference. The first criteria is that the pulse shape exhibits a zero crossing at the sampling point of all pulse intervals except its own. Otherwise, the residual effect of other pulses will introduce errors into the decision making process and the second criteria is that the shape of the pulses be such that the amplitude decays rapidly outside of the pulse interval [8]. The pulse shape used for transmission should have low bandwidth and also have no intersymbol interference (ISI). A sinc function has both these properties and hence can significantly increase spectral efficiency. However a system using a sinc function for pulse shaping is highly susceptible to timing jitter and phase error. This is one of the main drawbacks to be considered while designing practical pulse shaping filters. A rectangular wave filter is not as sensitive to timing jitter but needs a very high bandwidth. The raised cosine filter is commonly used in practical applications as it provides an optimal trade off between spectral efficiency and design complexity.

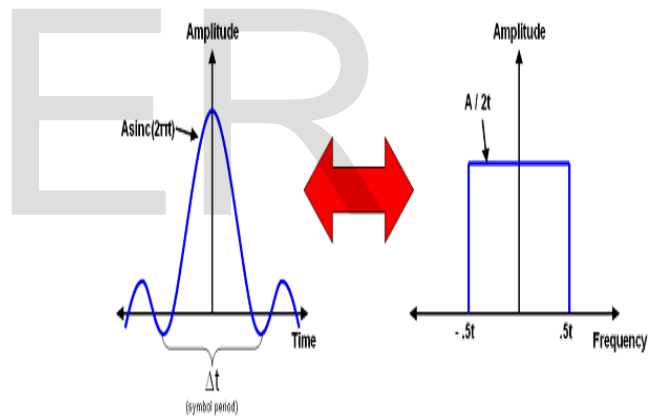


Fig. 2. Sinc Pulse Time & Frequency Response [9]

The sinc pulse is periodic in nature and is has maximum amplitude in the middle of the symbol time. In addition, it appears as a square wave in the frequency domain and thus can effectively limit a communications channel to a specific frequency range. The unbounded frequency response of the rectangular pulse renders it unsuitable for modern transmission systems. This is where pulse shaping filters come into play. If the rectangular pulse is not the best choice for band-limited data transmission, then what pulse shape will limit bandwidth, decay quickly, and provide zero crossings at the pulse sampling times. The Gaussian filter is used to solve this problem in a wide variety of modern data transmission systems.

In electronics and signal processing, a Gaussian filter is a filter whose impulse response is a Gaussian function. Gaussian filters are designed to give no overshoot to a step function input while minimizing the rise and fall time. It has the minimum possible group delay. It is a linear filter thus it is used as a smoother. The Gaussian filter is non-causal which means the filter window is symmetric about the origin. This gives an output pulse shaped like a Gaussian function. Mathematically, a Gaussian filter modifies the input signal by convolution with a Gaussian function; this transformation is also known as the Weierstrass transform. The one-dimensional Gaussian filter has an impulse response given by

$$g(x) = \sqrt{\frac{a}{\pi}} e^{-a \cdot x^2} \tag{3}$$

and the frequency response is given by

$$g(f) = e^{-\frac{\pi^2 f^2}{a}} \tag{4}$$

with  $f$  the ordinary frequency. These equations can also be expressed with the standard deviation as parameter

$$g(x) = \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{x^2}{2\sigma^2}} \tag{5}$$

and the frequency response is given by

$$g(f) = e^{-\frac{f^2}{2\sigma_f^2}} \tag{6}$$

In two dimensions, it is the product of two such Gaussians, one per direction:

$$g(x, y) = \frac{1}{2\pi\sigma^2} \cdot e^{-\frac{x^2+y^2}{2\sigma^2}} \tag{7}$$

where  $x$  is the distance from the origin in the horizontal axis,  $y$  is the distance from the origin in the vertical axis, and  $\sigma$  is the standard deviation of the Gaussian distribution. The Gaussian filter is non-causal which means the filter window is symmetric about the origin in the time-domain. No amount of delay can make a Gaussian filter causal, because the Gaussian function is never zero. Due to the central limit theorem, the Gaussian can be approximated by several runs of a very simple filter such as the moving average. Gaussian filter is used in GSM since it applies GMSK modulation.

### 3 DALUT Algorithm

Distributed Arithmetic (DA) plays a key role in embedding DSP functions in the Xilinx 4000 family of FPGA devices. Distributed Arithmetic, along with Modulo Arithmetic, are computation algorithms that perform multiplication with look-up table based schemes. This technique, first proposed by Croisier is a multiplier-less architecture that is based on an efficient partition of the function in partial terms using 2's complement binary representation of data. It is used for Multiplication operations to replace MAC operations. It uses look-up tables and accumulators instead of multipliers for computing inner products. So it provides multiplier less MAC operations. DALUT is an efficient method for computing inner products when one of the input vectors is fixed. The main operations required for DA-based computation of inner product are a sequence of lookup table (LUT) accesses followed by shift accumulation operations of the LUT output. DA-based computation is well suited for FPGA realization, because the LUT as well as the shift-add operations, can be efficiently mapped to the LUT-based FPGA logic structures. In FIR filtering, one of the convolving sequences is derived from the input samples while the other sequence is derived from the fixed impulse response coefficients of the filter. This behaviour of the FIR filter makes it possible to use DA-based technique for memory-based realization. It can be widely used in many DSP applications such as DFT, DCT, Convolution, and Digital filters.

#### 3.1 1-Tap FIR Filter

Distributed arithmetic is based on saving partial products in memories. Because the coefficients are known ahead of time, it is possible to pre-calculate the result of a multiplication. In this example, we are looking at a 1-tap FIR filter. The result of the multiplication is either  $0 \times \text{coef}$  or  $1 \times \text{coef}$ . Hence, the LUT will be initialized with 0 at location 0 and  $C0$  at location 1.

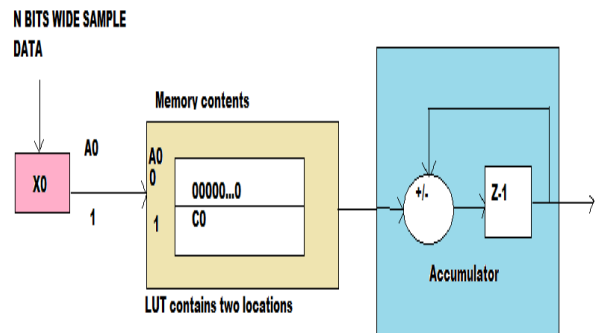


Fig. 3. FIR Filter (1-Tap)

The multiplier less distributed arithmetic (DA)-based technique has gained substantial popularity, Due to its high-throughput processing capability and increased regularity, results in cost-effective and area-time efficient computing structures.

### 4 Proposed Gaussian Filter Design

A continuous-time Gaussian filter has been developed with symbol time  $f_s = 1$  MHz The proposed Gaussian filter has been designed and simulated using Matlab [7].

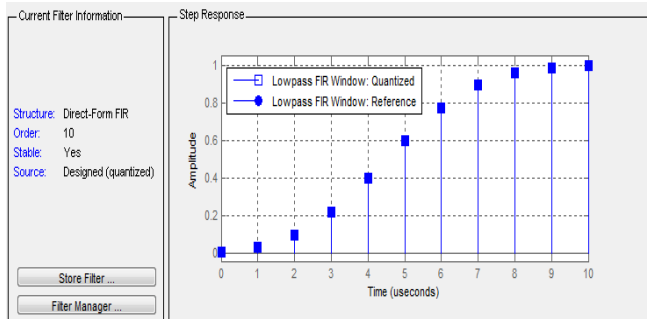


Fig. 4. Step Response of Gaussian filter

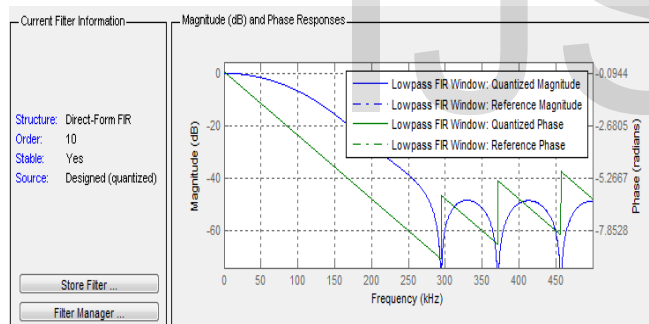


Fig. 5. Magnitude and Phase Response of Gaussian filter

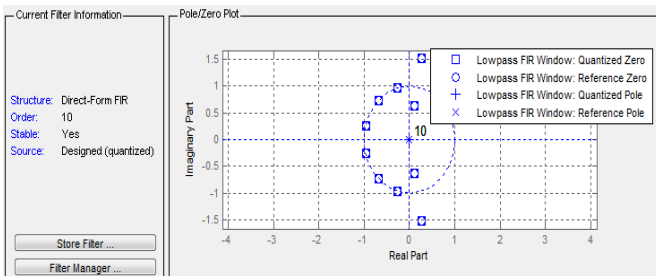


Fig. 6. Pole zero plot of Gaussian filter

### 5 Implementation Results and Discussion

The proposed pulse shaping Gaussian filter has been designed and implemented using DALUT algorithm. The multiplierless design of pulse shaping filter design has been synthesized and implemented on Spartan-3E based 3s500efg320-4 target device and simulated with modelsim simulator. The 12 bit input and output precision has been used to implement the proposed filter. The modelsim simulator has been used to simulate the proposed as shown in Figure 5. Table 1 shows the resource utilization of the proposed Gaussian filter.

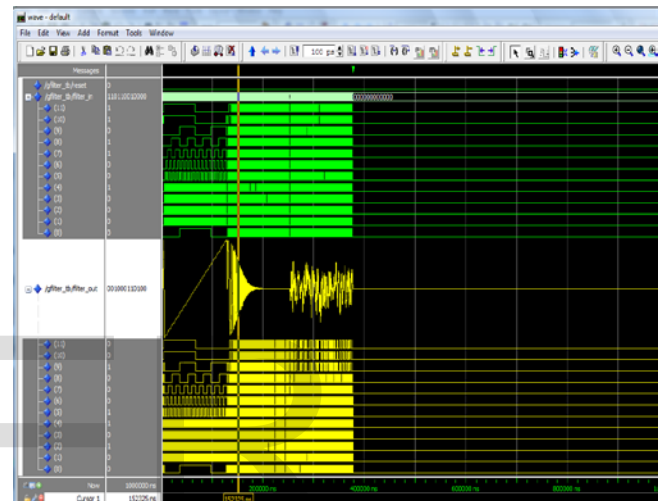


Fig. 7. Modelsim based Gaussian Filter Response

TABLE 1  
Resource Utilization

Device Utilization Summary (estimated values)			
Logic Utilization	Used	Available	Utilization
Number of Slices	220	4656	4%
Number of Slice Flip Flops	169	9312	1%
Number of 4 input LUTs	408	9312	4%
Number of bonded IOBs	27	232	11%
Number of GCLKs	1	24	4%

TABLE 2  
Comparison with Existing Designs

Logic Utilization and speed	Gaussian design[8]	Proposed Design	Available
Number of Slices	942	220	4656
Number of Slice Flip Flops	413	169	9312
Number of 4 input LUTs	1380	408	9312
Number of bonded IOBs	27	27	232
Minimum Period(ns)	74.984	6.073	-
Maximum Frequency(MHz)	13.336	116.306	-

The proposed DALUT algorithm based Gaussian filter can operate at a maximum operating frequency of 116.306 MHz with minimum period of 6.073 ns. The proposed multiplier less design of pulse shaping filter has consumed only 220 slices, 169 slice flip flops and 408 LUTs available on target device.

## 6 Conclusion

In this paper, a Gaussian pulse shaping filter has been presented to remove the ISI in wireless communication systems. The simulated results of proposed Gaussian filter show efficient rejection of ISI to provide high performance error free solution for wireless and mobile applications. The analysis and simulation is done using MATLAB. The simulated results of proposed Gaussian filter show efficient removal of errors. Finally developed filter has been implemented on target FPGA device using DALUT algorithm. The LUTs of target device have been efficiently utilized. The results have shown that proposed DALUT based design can operate with minimum period of 6.073 ns by consuming less resources in comparison to MAC algorithm based Gaussian filter. The proposed filter has consumed 220 slices, 169 flip flops and 408 LUTs available on target FPGA device.

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